Chapter 8

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

A **conservation law** is a physical principle that identifies some quantity that does not change with time.

Conservation of Energy:

The total energy in the universe is unchanged by any physical process:

total energy before = total energy after

Note that we have a precise definition of energy in physics!

A review might say that my favorite band (The Kills) play with a lot of energy, but… we're going to be more precise and mathematical (we are all scientists here!)

Some problems can be solved using either energy conservation *or* Newton's second law, so it always pays to consider both methods.

If both methods can be used to answer the question, think about which is easier to apply.

When time permits, solve the problem both ways.

It is good that both strategies give the same answer - if they didn't, we would have a problem (and not a homework problem)!

Forms of Energy

Energy comes in many different forms.

At the most *fundamental* level, there are only three types of energy, and only two that we care about in this course: energy due to motion (**kinetic energy**), and stored energy due to interaction or configuration of the system (**potential energy**).

Every form of energy can be understood as one or more of these types.

For now, we use energy conservation as a tool to understand the **translational** motion of objects, but we do not consider rotational motion or changes in the *internal* energy of an object.

We assume that these moving objects are perfectly rigid, so every point on the object moves through the same displacement.

Look under a powerful enough microscope at any object, and what do you see?

Translational kinetic energy:

$$K = \frac{1}{2}mv^2$$

v^2, so we don't worry about direction of v! Good, because energy is NOT a vector (even though we may use vectors to compute it)

Kinetic energy for more than one object

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \ldots = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2$$

Units of Joules (J)
= 1 Nm = 1 kg m^2/s^2

Kinetic energy cannot be negative, and sums simply as a normal scalar

Gravitational Potential Energy When Gravitational Force Is Constant

Toss a stone up with initial speed v_i. From the standpoint of energy conservation, where did the stone's initial kinetic energy go?

If total energy cannot change, it must be "stored" somewhere.

Stored energy due to the interaction of an object with something else (here, Earth's gravitational field) that can easily be recovered as kinetic energy is called **potential energy** (symbol U).

Other Forms of Potential Energy

Other kinds of potential energy include **elastic potential energy** and **electric potential energy**.

Forces that have potential energies associated with them are called **conservative forces**.

Not every force has an associated potential energy. For instance, there is no such thing as "frictional potential energy."

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

Not happy with integrals yet? It's OK…

Units of Joules = 1 Nm

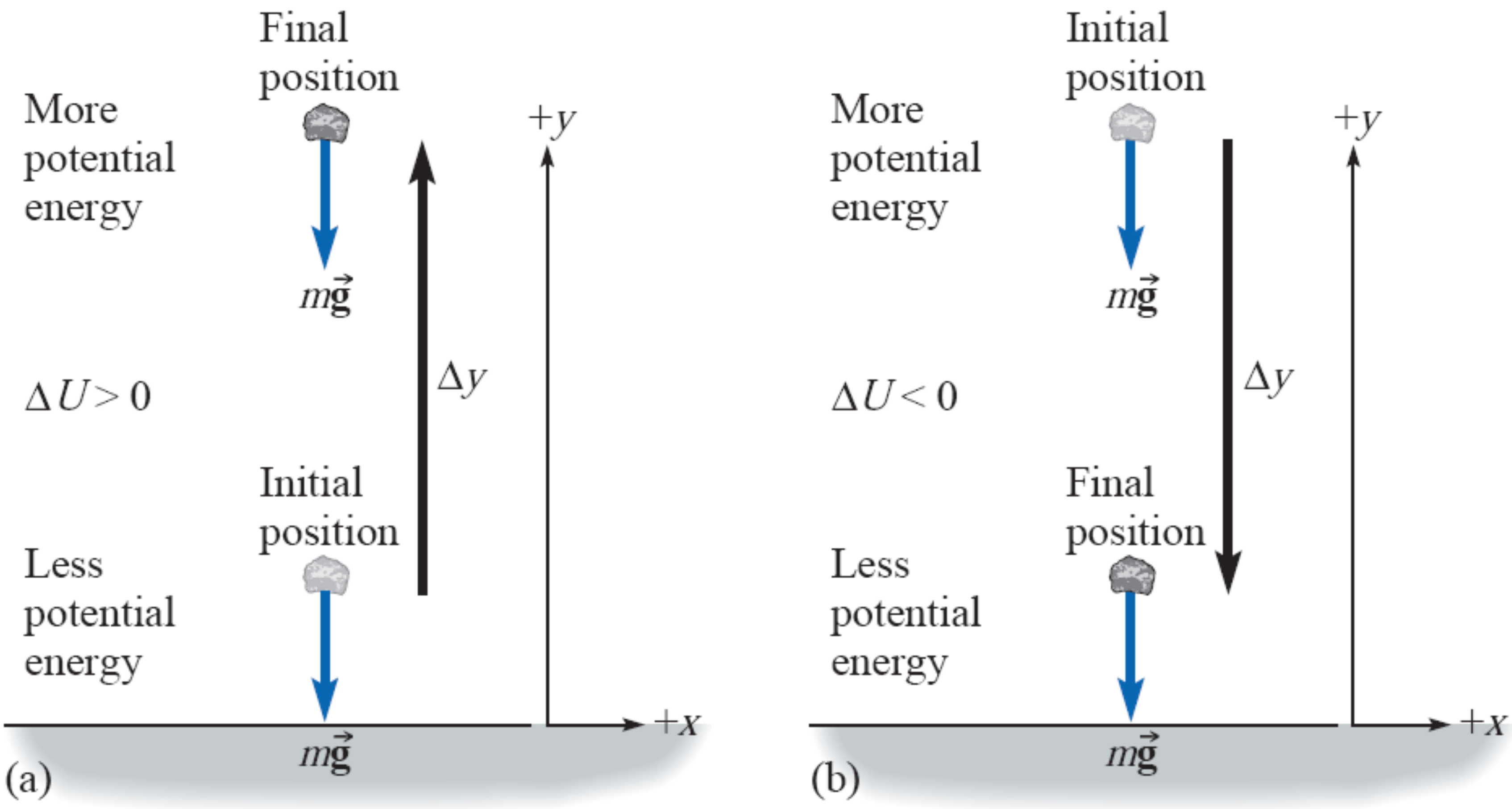
Gravitational potential energy

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

$$\Delta U_G = -\int_{y_i}^{y_f} -mgdy = mg(y_f - y_i) = mg\Delta y$$

F_G points in negative y direction!

Checking negative signs



Let's make sure to take a close, close look at this

Gravitational Potential Energy When Gravitational Force Is Constant

The stone's loss of kinetic energy ($\Delta K = -mg\Delta y$) is accompanied by an increase in gravitational potential energy ($\Delta U = +mg\Delta y$).

(This holds *even if the object does not move in a straight-line path, and it doesn't depend on where it was in between, only its initial and final y values!*)

The sum of the kinetic and potential energies ($K + U$) is called the **mechanical energy**.

Conservative forces such as gravity do *not* change the mechanical energy; they just change one form of mechanical energy into another.

When only conservative forces are acting, mechanical energy is conserved.

$$(K_i + U_i) = E = (K_f + U_f)$$

Choosing Where the Potential Energy Is Zero

For gravitational potential energy in a uniform gravitational field, we usually choose the potential energy to be zero at some convenient place: on the floor, on a table, or at the top of a ladder. We have this freedom and flexibility to choose our coordinate system, and we only care about **CHANGES** in U

After assigning $y = 0$ to that place, the potential energy at any other place is $U = mgy$.

Do we see why that is?

The expressions for gravitational potential energy we derived apply when the gravitational force is *constant* (or nearly constant). This is true near the Earth's surface (**WHY?**), but not if you go far from it

Recall that the magnitude of the gravitational force that one body exerts on another is

$$F = \frac{Gm_1m_2}{r^2}$$

where *r* is the distance between the centers of the bodies, and that

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

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Here, "x" is along the direction of motion, which is the radial direction. But the field points objects back **towards** the body of mass, towards smaller r, so the force is in the -rhat direction. So …

$$\Delta U = -\int_{r_i}^{r_f} -\frac{Gm_1m_2}{r^2} dr$$

$$\Delta U = Gm_1m_2 \int_{r_i}^{r_f} \frac{1}{r^2} dr = -Gm_1m_2 [1/r]_{r_i}^{r_f}$$

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$$\Delta U = Gm_1m_2(\frac{1}{r_i} - \frac{1}{r_f})$$

Let's check the sign. What happens if you move away from the main body? Towards it?

The corresponding expression for gravitational potential energy in terms of the distance between two bodies is

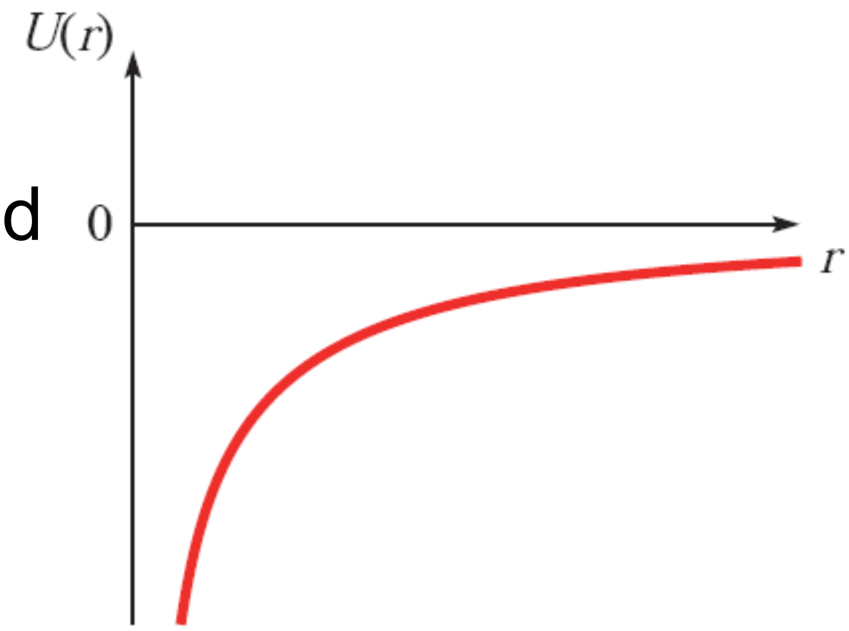
Gravitational potential energy:

Not happy with integrals yet? Then just use this and review it when you are:)

$$U = -\frac{Gm_1m_2}{r}$$

(assign $U = 0$ when $r = \infty$)

Note that we have assigned the potential energy to be zero at infinite separation (U = 0 when $r = \infty$).



On gravitational potential energy:

$$U = -\frac{Gm_1 m_2}{r} \qquad (\text{assign } U = 0 \text{ when } r = \infty)$$

U = mgy (assign U = 0 when y = 0)

As before, the top one is more correct, but when changes in distance from the Earth's center are small compared to the radius of the Earth, the second one is much easier to use!

The force exerted by a spring is also **conservative**, and we can associate a potential energy with it.

The kind of potential energy stored in a spring is called **elastic potential energy**.

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

$$\Delta U = -\int_{x_i}^{x_f} -kx dx$$

$$\Delta U = k\int_{x_i}^{x_f} x dx$$

$$\Delta U = \frac{1}{2}k(x_f^2 - x_i^2)$$

Can be positive or negative, but stretching and compressing the spring by the same amount gives the same potential energy change!

Elastic potential energy stored in an ideal spring:

$$U_{\text{elastic}} = \tfrac{1}{2}kx^2$$

$$U = 0 \text{ when } x = 0 \text{ (relaxed spring)}$$

Conservation of Energy with More than One Form of Potential Energy

When applying conservation of energy using $\Delta K = \Delta U$, ΔU must include the change in all forms of potential energy. For example, with the two forms of potential energy we discussed so far,

$$\Delta U = \Delta U_{\text{grav}} + \Delta U_{\text{elastic}}$$

Interesting question: Ignoring air resistance, find the minimum initial speed a projectile must have at Earth's surface if the projectile is to escape Earth's gravitational pull.

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Stepping back

The only force acting on the projectile is gravity, so the mechanical energy is constant. To escape, the projectile must have enough initial kinetic energy so that it can reach an unlimited distance from Earth.

Let's discuss why that is!

Solution

$$K_i + U_i = K_f + U_f$$

What happens if $K_f > 0$?

$$\frac{1}{2}mv_i^2 + \left(-\frac{GM_E m}{R_E}\right) = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + \left(-\frac{GM_E m}{R_E}\right) = 0 + 0$$

as $r_f \to \infty$, $U_f \to 0$

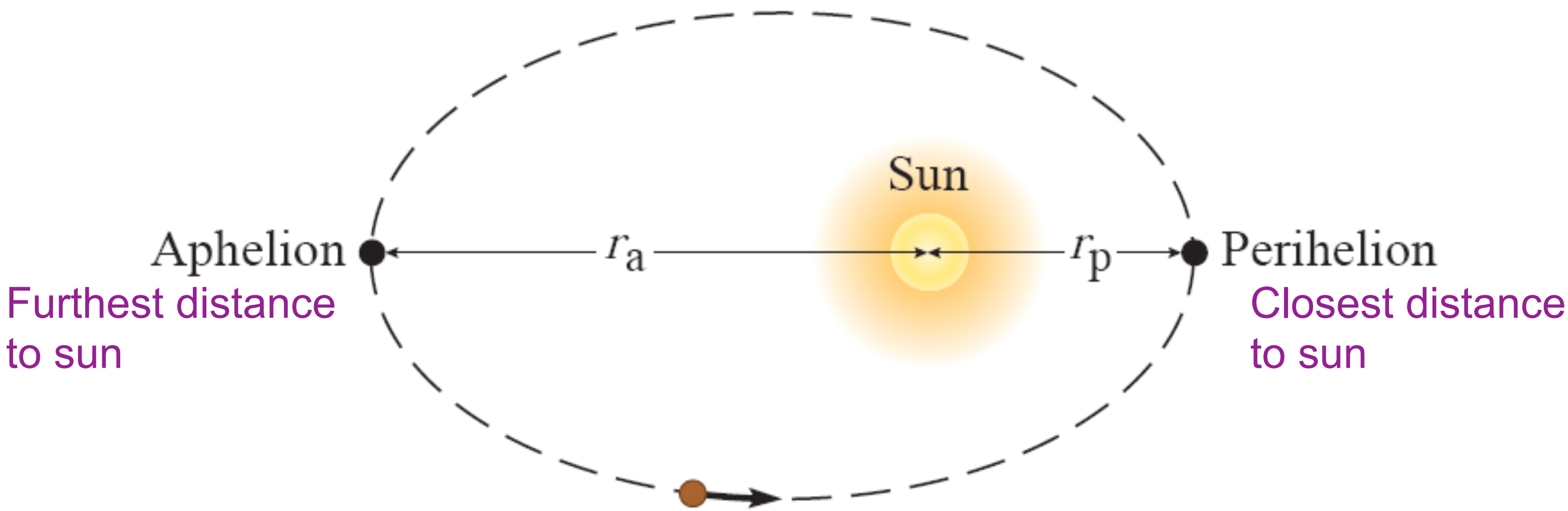
$K_f = 0$

$$\frac{1}{2}\cancel{m}v_i^2 = \frac{GM_E \cancel{m}}{R_E} \quad \Rightarrow \quad v_i = \sqrt{\frac{2GM_E}{R_E}} = 11.2 \text{ km/s}$$

GROUP WORK TIME!

https://forms.gle/CfGiKcNL87zKmo866

The orbit of the planet Mercury around the Sun is an ellipse. At its perihelion (4.60×10^7 km), its orbital speed is 59 km/s. What is its orbital speed at aphelion (6.98×10^7 km)?



In the design of a roller coaster, is it possible for any hill of the ride to be higher than the first one? If so, how?

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Yes, if the first hill is reached with some KE > 0!

A 1700 kg blue car traveling due north at 30 m/s passes a 1200 kg red car traveling due south at 20 m/s.

a) According to someone in the blue car, what is the kinetic energy of the red car?

b) According to someone in the red car, what is the kinetic energy of the blue car?

A 0.430 kg soccer ball is kicked at an initial speed of 34.0 m/s at an angle of 35 degrees with respect to the horizontal. What is the kinetic energy of the soccer ball when it has reached the apex of its trajectory?

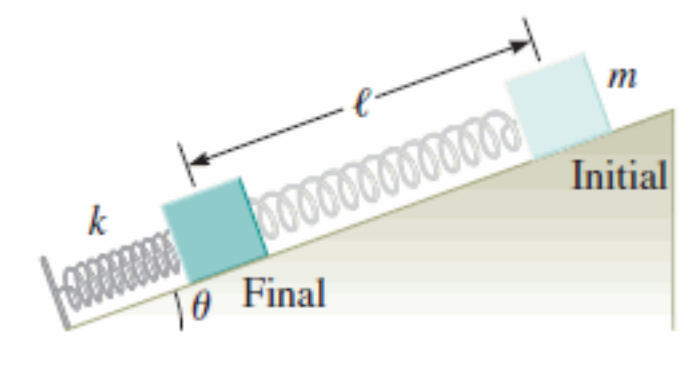
A ball of mass 2.5 kg is given an initial velocity upwards of 15 m/s. What is the change in the gravitational potential energy of the ball from its initial height to when the ball is at the peak of its motion?

The average distance between the Earth and the Sun is about 1.5 x 10^11 m, whereas the average distance between the Earth and Mars is about 2.3 x 10^11 m. Assume the usual convention that the gravitational potential energy referenced to the Earth is zero when infinitely far away.

a) What is the gravitational potential energy of the Earth-Sun system?
b) What is the gravitational potential energy of the Earth-Mars system?
c) Describe how these indicate that the Sun has a greater effect on the Earth’s motion than Mars does.

A spring of spring constant k lies along an incline as shown in the Figure. A block of mass m is attached to the spring. The spring compresses, and the block comes to rest as shown. Find an expression for:

a) The change in the system's spring potential energy in terms of the parameters given in the Figure.
b) The change in the system's gravitational potential energy in terms of the parameters given.
c) The change in total potential energy of the system



A falcon is soaring over a prairie, flying at a height of 45.0 m, with a speed of 12.9 m/s. The falcon spots a mouse running along the ground and dives to catch its dinner. Ignoring air resistance, and assuming the falcon is only subject to the gravitational force as it dives, how fast will the falcon be moving the instant it is 5.00 m above the ground?

A man unloads a 5.0-kg box from a moving van by giving it an initial speed of 1.3 m/s down a ramp that makes an angle of 20 degrees with the horizontal. It slides with negligible friction for 2.7 m along the ramp before reaching the man's helper. What is the speed of the box as the helper catches it?

An intrepid physics student decides to try bungee jumping. She obtains a cord that is 9.00 m long and has a spring constant of 5.00 x 10^2 N/m. When fully suited, she has a mass of 70.0 kg. She looks for a bridge to which she can tie the cord and step off. Determine the minimum height of the bridge that will allow her to stay dry (that is, so that she stops just before hitting the water below). Ignore air resistance.

A 50.0-g toy car is released from rest on a frictionless track with a vertical loop of radius R. The initial height of the car is h = 4.00R.

a) What is the speed of the car at the top of the vertical loop?
b) What is the magnitude of the normal force acting on the car at the top of the vertical loop?

A chair has 6 identical springs, each with spring constant 1.5 x 10^3 N/m. What is the stored elastic potential energy in the six-spring system if your friend with mass 65.0 kg sits on the chair?

At the start of a basketball game, the referee tosses a basketball straight into the air by giving it some initial speed. After being given that speed, the ball reaches a maximum height of 4.25 m above where it started. Using conservation of energy find:

a) The ball’s initial speed
b) The height of the ball when it has a speed of 2.5 m/s

Rachel is on the roof of a building h meters above ground. She throws a heavy ball into the air with a speed v, at an angle θ with respect to the horizontal. Ignore air resistance.

a) Find the speed of the ball when it hits the ground in terms of h, v, g and θ
b) For what value(s) of θ is the speed of the ball greatest when it hits the ground?

A projectile with mass 500 kg is launched straight up from the Earth's surface with an initial speed v. What magnitude of v enables the projectile to just reach a maximum height of 5 R_E measured from the center of the Earth? Ignore air resistance and friction

The length of a spring increases by 7.2 cm from its relaxed length when a mass of 1.4 kg is hanging in equilibrium from it

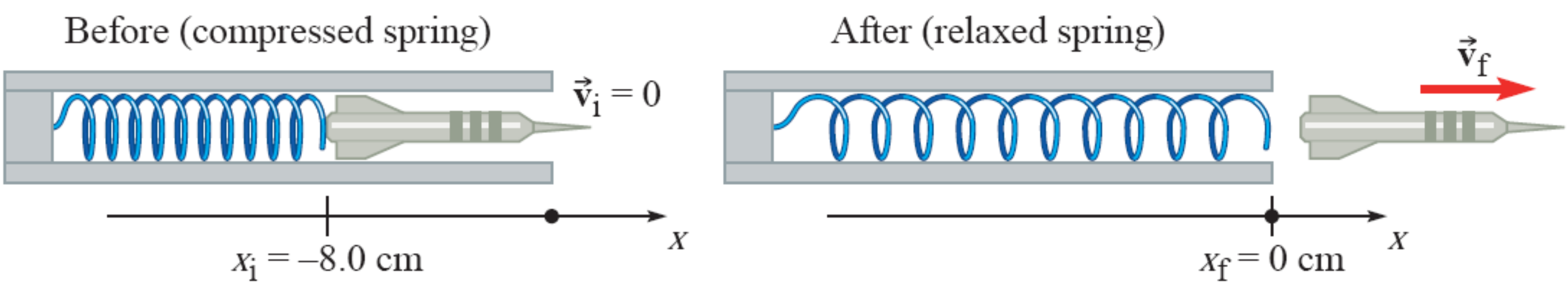
a) What is the spring constant?
b) How much elastic potential energy is stored in the spring?
c) A different mass is suspended and the spring length increases by 12.2 cm from its relaxed length to its new equilibrium position. What is the second mass?

You throw a baseball straight up in the air at a velocity of 33 m/s. How high up does it go, if we ignore all air resistance and friction? Do we need to know its mass?

You toss a baseball up in the air at a velocity of 33 m/s. It comes back down and lands on a spring with spring constant k = 1000 N/m. How much is the spring compressed from its nominal length? The mass of a baseball is 0.145 kg.

In a dart gun, a spring with k = 400 N/m is compressed 8.0 cm when the dart (mass m = 20.0 g) is loaded.

What is the muzzle speed of the dart when the spring is released? Ignore friction.



Let's consider a different dart gun (with a new spring constant). When the spring is compressed to a position x_i the dart is launched to a speed v_f.

Afterwards, the dart is reconnected and the experiment is repeated. How far would you have to compress the spring to launch to a speed three times as big as before?

Let's consider a different dart gun (with a new spring constant). When the spring is compressed to a position x_i the dart is launched to a speed v_f.

Afterwards, a new dart with a different mass is reconnected and the experiment is repeated. If the spring is compressed to the same position, what must the mass of the new dart be for it to launch to a speed three times as big as before?